Well-behaved epireflections for Kan extensions and models of sketches

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Abstract.

Any adjunction produces an epireflection in a canonical way, provided there exists a prefactorization system which factorizes all of its unit morphisms through epimorphisms. The stable units property, for the so obtained epireflections, is thereafter equivalently restated in such a manner it is easily recognizable in the examples. Furthermore, having stable units, there is a strong but quite simple sufficient condition for the existence of an associated monotone-light factorization. These results have proved to be fruitful when the domain of the left adjoint, in the ground adjunction, is either a functor category, or a category of internal structures, like categories or groupoids. For instance, in the former case, if \( K : \mathcal{B} \to \mathcal{A} \) is a functor such that the image of the objects in \( \mathcal{B} \) is a cogenerating set of objects for \( \mathcal{A} \), then the right Kan extensions adjunction \( \text{Set}^K \dashv \text{Ran}_K \) induces necessarily an epireflection with stable units and a monotone-light factorization. The monotone-light factorizations obtained for the latter internal case are just the restrictions of the ones which arise in the former external case. Why this is so will be explained by considering sketches, i.e., categories with distinguished families of cones, which leads us to higher-category theory as an obvious source of examples.

References


