The Evolution of Our Local Cosmic Domain: Effective Causal Limits

G. F. R. Ellis[1] and W. R. Stoeger[1,2]

[1] *Mathematics Department, University of Cape Town, Rondebosch, Cape Town 7701*
[2] *Vatican Observatory Research Group, Steward Observatory, The University of Arizona, Tucson, AZ 85721.*

15 May 2009

ABSTRACT

The causal limit usually considered in cosmology is the particle horizon, delimiting the possibilities of causal connection in the expanding universe. However it is not a realistic indicator of the effective local limits of important interactions in spacetime. We consider here the *matter horizon* for the Solar System, that is, the comoving region which has significantly contributed matter to our local physical environment. This lies inside the *effective domain of dependence*, which (assuming the universe is dominated by dark matter along with baryonic matter and vacuum-energy-like dark energy) consists of those regions that have had a significant active physical influence on this environment through effects such as matter accretion and acoustic waves. It is not determined by the velocity of light c, but by the flow of matter perturbations along their world lines and associated gravitational effects. We emphasize how small a region the perturbations which became our Galaxy occupied, relative to the observable universe – even relative to the smallest- scale perturbations detectable in the cosmic microwave background radiation. Finally, looking to the future of our local cosmic domain, we suggest simple dynamical criteria for determining the *present domain of influence* and the *future matter horizon*. The former is the radial distance at which our local region is just now separating from the cosmic expansion. The latter represents the limits of growth of the matter horizon in the far future.

Key words: cosmology: theory

1 INTRODUCTION

Causal limits in cosmology are usually taken to be given by the past light cone. This paper aims to introduce a more nuanced view of these limits, based on timelike world lines.

In probing the universe as a whole – as a single object of study – we peer out to collect and interpret data arriving at the speed of light from its farthest reaches, and therefore from its earliest epochs. Much of observational cosmology relies on precise measurements of the cosmic microwave background radiation (CMB), which emanates from the last-scattering surface where it last interacted significantly with matter some 13.7 billion years ago. Because of the opaqueness of the early universe, this is the most distant region from which we can receive electromagnetic signals, a scale of at least 10^{29} cm. However, these signals are difficult to detect and have negligible influence on the universe today; while they represent absolute limits to observation, they themselves do not exert a significant influence on present-day conditions in local regions. The same is true for other particles that may arrive from very distant regions, such as neutrinos. Even though the global expanding cosmic environment from Planck-era exit until today has been important for establishing conditions of large-scale near-homogeneity and consequent relative dynamical isolation of local regions, large-scale global influences have not, since inflation, in any other way actively affected evolution on present comoving scales larger than about 100 Mpc.

This analysis presumes that the present cosmological picture is more or less correct – i.e that the dominant mass-energy constituents in the universe are cold dark matter and vacuum-like dark energy, along with baryonic matter and radiation. We shall, as standard, refer to this as the ΛCDM model, and will assume that it, or something not too much different from it, is a correct description of the universe's principal mass-energy components. What follows is only true if there are no exotic components of the universe (dynamical dark energy, cosmic strings etc...) with either relativistic peculiar velocities, relativistic speeds of sound, very large Compton wavelengths or even more extreme ideas such as variable speed of light theories or deviations from Einstein gravity. There is always the possibility that our lack of understanding about dark matter, dark energy, and

the lower-multipole anomalies in the CMB may be signaling the presence of such exotic mass-energy constituents, or a large-scale modification of the gravity, which would alter our assessment. But these and other unsolved cosmological problems may also turn out to be understood in terms completely consistent with the causal mass-energy picture we now have, and we will from here on proceed on that basis. And on that basis, most of what we can see by astronomical observations has negligible effect on us. If one of the more exotic possibilities were to turn out to be true, what follows would need modification in accord with those discoveries.

There is therefore some interest in investigating the space-time region that *does* make a significant difference to local conditions[1]. We can do so reversing our perspective and looking carefully at the cosmological history of our own local domain and the 'geological' evidence it contains about the early universe. We distil cosmological significance from it, for example, by relating the current abundances of elements in our neighbourhood to baryosynthesis and nucleosynthesis close to our world line in the very distant past, long before the last scattering surface was established. Our local domain has a history stretching back through the epochs of recombination, the Hot Big Bang radiation dominated era, and inflation, to the end of the Planck era and the start of the classical universe. And it has a future related to nearby galaxies and clusters of galaxies – all the large-scale "perturbations" with which it will eventually interact as the universe evolves towards its distant future.

How then can we characterize causality related to our local cosmic domain? Specifically: how large is the region in the past which significantly contributed to conditions in our local physical system? How large is the region which presently interacts with it? How large is the region that will affect it in the future? Our primary aim is to define precisely effective domains of dependence, that is, the portion of the initial data which makes a significant difference to the evolution of our local systems – our Galaxy and the Solar System – from some initial time in the past up to the present time. This depends on the epoch at which we evaluate it. We would like to know how large it was at recombination, at the end of inflation, and prior to inflation; and even further back at its very origins.

1.1 The effective horizon

The basic limit of possible causal influence in a space time (the past light cone) leads to the absolute limit of causal influence in cosmology as we extend our past light cone to the origin of the universe. This absolute limit is the *particle horizon*, defined as the limiting sphere of the most distant matter that could have influenced us up to the present time by any interaction proceeding at speeds less than or equal to the speed of light (Rindler 1957, Hawking and Ellis 1973, Tipler, Clarke and Ellis 1980); most studies of causality in cosmology focus on these causal limits. However this is not an adequately nuanced representation of the domain of influence significantly affecting our local cosmic neighborhood, because, as far as we have been able to determine on

the basis of the ΛCDM model, the influences that make a significant difference to conditions in the Solar System and our Galaxy (the Milky Way, or *MW*) are mediated through massive rather than massless particles; and these particles travel at very low speeds relative to the cosmic rest frame. This is related to the fact that the characteristics for scalar perturbations of pressure-free matter are timelike world lines (Ehlers, Prasanna and Breuer 1987), and these are far more important in cosmological history (as presently understood) than vector or tensor perturbations.

Thus, in the ΛCDM universe model, and in our universe insofar as it is described by that model, effective causal limits are based on timelike world lines of matter and associated local gravitational fields, rather than on influences travelling on null geodesics. At any finite time t, these worldlines determine the *Effective Domain of Influence*, which is the limiting sphere of matter that can have had a significant effect on our local region from time t to the present time t_0, with comoving radius $r_{rd}(t, t_0)$. Taking the limit of the time t towards the start of the universe defines our *Effective Horizon*, which is the analogue of the particle horizon: it is the limit of the spacetime region that contains all matter and events that have contributed significantly to the specific history and characteristics of our local domain since the start of the universe. It is thus the boundary of the total set of events that have made a noticeable difference to what is happening here and now, through the effects of the massive particles that have in fact determined local conditions on and near the Earth. Our lives have would have been significantly different if conditions there had been altered.

1.2 The matter horizon

From this perspective our local domain of influence back to decoupling consists of the matter in density perturbations which evolved into our Galaxy, including nearby matter perturbations with which they merged or interacted. If we take our Galaxy as our present local cosmic domain – as the smallest object which we can be reasonably sure has an identifiable interaction history with us without significantly involving any larger scales– we can consider its history as it passes through the inflationary process, reheating, the radiation dominated epoch, the radiation-matter equality transition, etc., going all the way back to when it was a cluster of quantum fluctuations just after the Big Bang.

This history reveals that the actual domains of influence affecting local physical systems in the universe today since recombination at $z_{rec} \approx 1100$, are for all practical purposes, confined to the bundle of world-lines gradually bringing matter together to form our Galaxy and our Solar System. The significant factor to be considered at times since decoupling is the effective local velocities by which matter was added to the Galaxy from nearby regions. Indeed one can claim, within the present provisionally accepted ΛCDM model, that the main locally significant causal domain – the region from which the vast majority of constituent matter originated[2] – was characterised by the same comoving matter even long before that, being relatively unchanged since

---

[1] Colloquially: which part of space time makes a real difference to you, as you walk down the street or drive a car?

[2] Leaving aside neutrinos and other possible weakly interacting matter components, as well as cosmic rays from distant galaxies,

the decoupling of the dominant cold-dark matter (CDM) at z_{CDM}. It is all contained in a sphere of present comoving size about 2 Mpc, i.e. 6×10^{25}cm.

We call this bounding sphere, the *matter horizon*. This horizon, then, is the limit beyond which no significant matter has been contributed to our local domain today. It obviously evolves and is growing at present, as it has in the past when matter was accreted onto our local high-density region. Its starting point is at the end of inflation, when the high vacuum left by the rapid inflationary expansion was filled with matter created by decay of the inflaton field. This is the originating event for the matter which exists in and on the earth today. The original matter (quarks and electrons?) has since been processed in many ways, in particular through baryosynthesis and nucleosynthesis; but these were local processes that did not significantly alter the domain bounded by the matter horizon.

At earlier times, before decoupling at t_{rec}, the velocity of sound c_s at which acoustic oscillations established correlations among density perturbations within a somewhat larger region, and so allowed a larger domain to influence local densities even though no exchange of matter took place between the relevant regions. That is why the effective domain of influence can grow to be larger than indicated by the matter horizon, at times before decoupling.

1.3 The present domain of influence and the future horizon

We shall find it natural to define also a *future matter horizon* and a *present domain of influence*, specifying the radius from our position beyond which collapse towards us will never occur in the future, and the radius beyond which collapse is not occurring now, respectively. It is easy to confirm that these are also much less than both the particle horizon and the visual horizon.[3] The very fact that distant galaxies are now expanding away from us – and most of them will continue to do so forever – is secure evidence of this.

In what follows, we aim to determine the sizes of these effective domains for the MW in a physically rigorous and potentially measurable way. Our discussion can be easily "scaled up" to a larger region – e.g. the Local Group – simply by considering the total mass contained in it, as we do here for the MW, and tracing its history back to recombination and before.

2 THE PAST: BACK TO RECOMBINATION (PHOTON DECOUPLING)

It has become clear that the dominant galaxy formation processes in our universe are bottom-up – smaller galaxies formed first, and then gradually agglomerated into larger

galaxies and clusters of galaxies. This is the principal basis for choosing the worldlines of the matter that made up our Galaxy as the cosmological domain of influence for the solar system, and then focusing on its likely cosmological history all the way back to the Planck era. This choice is strongly supported by the fact that the local cosmological environment in which our Galaxy finds itself is both noticeably underdense relative to nearby regions of our local supercluster, hence is not accreting matter from much farther out, and is surprisingly quiescent as a local peculiar velocity field (within $5h^{-1}Mpc$ around the MW) (see Klypin, et al., 2003, and references therein).[4] Despite this there is continual accretion of material on smaller length scales, including accretion of much smaller systems onto the MW (Cho 2008; Ibata and Lewis 2008), leading to what has been referred to as our local "cosmic web." On a much larger scale, there is Virgocentric infall. These discoveries strongly support the bottom-up account of galaxy assembly from a small local region around the galactic core of the MW.

Considering that all the matter presently constituting the MW was in the same narrow tube of timelike geodesics all the way back to recombination at $z_{rec} \approx 1100$, what was the length scale embracing this bundle of world lines? The condition for the dynamical importance of nothing larger than this bundle is certainly fulfilled, since the effective pressure –and therefore the sound speed – of the matter in the universe is zero from recombination/decoupling to the present, and the strengths of any other possible long distance influences – electromagnetic or gravitational waves, or Weyl tensor components from sources outside that region – are insignificant compared to local effects (Cox 2007). This is due to the large-scale almost-FLRW character of our universe, and to the relative weakness and at least r^{-2} fall-off of these interactions. The effective domain of influence for the MW as an evolving local system, then, is determined by the peculiar velocities of the material in the local gravitational field of the growing density fluctuations that eventually merge to form the MW. This defines the sphere of matter which has had significant influence on conditions here at the present time; in particular, it characterizes the matter out of which the MW is composed.

Estimating the size of this domain of influence at z_{rec} is relatively simple. At the time of recombination, just when the anisotropies of the CMB were imprinted, all the matter in the MW would have been in a localized cluster of nearby density perturbations just slightly more dense than the background at that time, by 1 part in a 100,000 (the density perturbations, as we know from CMB data, were about $\delta \approx 10^{-5}$ on a broad range of scales). If we take the total mass of the MW to be $7 \times 10^{11} M_\odot$ within 100 kpc (Binney and Merrifield 1998) and assuming $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70$ km/sec/Mpc, a simple continuity argument shows that the MW perturbations occupied a volume of radius 1.46 kpc at recombination, $z_{rec} = 1100$. This is equivalent to a comoving length scale (relative to the present time) of $d_{doi}(t_0, t_{rec}) = 1.6$ Mpc.

Nothing on larger scales has had a significant physical effect on conditions in our cosmic neighborhood at the

---

which – in accord with present evidence – we assume have little effect on the evolution of our local domain.

[3] The visual horizon is the limiting sphere of matter beyond which we cannot receive any unimpeded electromagnetic signals. It is defined by the matter world lines extending to the future from the intersection of our past light cone with the last-scattering surface. We have not yet received any direct information via any radiation from world lines beyond these.

[4] Here and elsewhere in this paper $h \equiv H_0/100$ km/sec/Mpc.

present time.[5] The present comoving size of the particle horizon in an Einstein de Sitter universe, which is essentially the size of the visual horizon in that universe, is $d_{phE_dS}(t_0) \simeq (1/3) \times 10^4$ Mpc, so the effective domain of influence d_{doi} at that time is very much less than this: $d_{doi}(t_0, t_{rec})/d_{phEdS}(t_0) \simeq 5 \times 10^{-4}$. In an inflationary universe, the particle horizon is very much bigger than the visual horizon (everything we can see), and the ratio is enormously smaller. The change from null to timelike causation radically redefines the domains of effective causal importance in cosmology.

It is enlightening to ask two simple questions about this result. The first is: What angular size would such a cluster of perturbations subtend on the last-scattering surface for a distant cosmic observer who was trying to resolve it as a fluctuation in her CMB data at the present cosmological time? The answer is that it would subtend only about 0.6 arcmin, or a spherical harmonic index $l \approx 5370$, and considerably less than the thickness of the last-scattering surface itself. The on-going ACT survey will probe this angular scale and smaller, but a detailed analysis of decoupling dynamics would be needed to see if traces of these inhomogeneities would remain in the CBR data.

But what are the conditions for such a perturbation to be present at that time? Isn't it small enough to be wiped out by photon-diffusion during decoupling? If the only form of matter present were baryons, then, yes, a comoving perturbation of 1.6 Mpc would indeed be dissipated by that process (Kolb and Turner 1990, p. 355, equation (9.97)). For a FLRW background cosmology with no CDM and $\Omega_b = 0.04$, for instance, perturbations on co-moving length scales smaller than about 30 Mpc would be dissipated by photon diffusion. However, if the dominant matter in the universe is non-baryonic, as seems to be the case – there is about 6 or 7 times more cold dark matter (CDM) in the universe than baryons – the picture changes completely. Then the CDM perturbations decoupled long before recombination and began to grow. During the decoupling of the baryons from the photons they would not be markedly affected by photon diffusion, and the baryons will begin to fall rapidly into those CDM potential wells tracing the CDM perturbations themselves. We shall return in the next section to discuss the decoupling and evolution of these crucial CDM fluctuations. The baryon dynamics after decoupling is purely gravitational and thus dominated by DM.

A second question about the early growth and history of our MW galaxy is: How large would the perturbations have been that eventually led to the MW, at recombination? How big were the individual pre-MW size lumps that existed on the LSS? [6] There was undoubtedly a cluster of significantly smaller perturbations, which were incorporated into the MW

over time, eventually attaining a comoving size of 1.6 Mpc (and so a physical size then of $1.4 Kpc$). A number of observational and theoretical studies have indicated (Carlberg, et. al. 2000; Murali, et al. 2002), for instance, that galaxies of the size of our MW have grown by as much as factors of 2 to 3 by mergers and accretion since $z = 1$ (about 7 Gyr for our cosmology). The amount of growth by these two processes before then would have been even more significant. Thus, the individual perturbations which combined to constitute the MW were undoubtedly originally much smaller than the 1.6 Mpc figure, and so even more difficult to resolve in the CMB anisotropy spectrum: they would subtend less than 0.3 arcmin, or a spherical harmonic index greater than $l \approx 10,740$. But we do not know what the mass-spectrum of these individual perturbations was.

3   THE PAST: FROM INFLATION TO RECOMBINATION

The question now is how far back did the domain of influence remain so small in comoving terms. It would have remained small as long as CDM dominated the dynamics, and CDM and baryons were decoupled from the radiation. But how do we know that there were CDM perturbations already growing before recombination allowed the baryons to fall into them? And what are the conditions constraining their size and determining their lower-end cutoff?

As regards the first question, we have nearly incontrovertible evidence from primordial nuclear abundances together with velocity dispersion measurements in clusters of galaxies, rotational curves in spiral galaxies, etc., that there must be a much larger percentage of matter in cold nonbaryonic particles than in baryons. Furthermore, this is consistent with the result that it would be impossible for the density fluctuations we see at last scattering (in the CMB) to grow into the structures we observe now if only the baryons were involved. There is not enough time. As we indicated above, there would not have been perturbations on comoving scales less than about 35 Mpc (because of photon diffusion at decoupling).

Thus the standard understanding is that cold dark matter (CDM) perturbations are crucial to structure formation in the universe, essentially dominating and accelerating the growth of baryonic perturbations after decoupling. It is the origin and growth of these CDM perturbations ancestral to the MW and to other galaxies that we need to focus upon. The smallest systems today which are dominated by primordial dark matter are dwarf irregular galaxies, with mass of the order of $2 \times 10^7 M_\odot$. These would originate in the comoving range of, say 0.05 to 1.0 Mpc, and would undergo mergers and accretion as perturbations – and later as galaxy haloes – with the expansion and cooling of the universe.

The answer to the second question depends on the epochs that occurred in the past. Would not the region of importance have been determined by the speed of light before decoupling? No, because the universe was opaque then: the mean free path for photons was very small. No influences, except gravitational waves, could travel at the speed of light, even when the universe was radiation dominated. The way this worked out varied with epoch and with wavelength.

---

[5] If the MW has 3 times the amount of matter quoted here, as suggested by some references, e. g. Peebles 1992, then the comoving size of the MW region at recombination will be a little larger – a radius of about 2.3 Mpc.

[6] We refer here to perturbations *like* those that led to the MW. We cannot see the MW perturbations themselves, no matter what size they were, as they were where *our past world line* intersected the LSS, nowhere near where our past light cone intersects it.

3.1   The relevant epochs

The relevant epochs before decoupling are as follows, in the order in which they occurred:

- Before inflation (a quantum gravity domain?).
- The inflation epoch (before reheating).
- Reheating (the end of inflation) to CDM decoupling.
- CDM decoupling to matter domination.
- Matter domination to radiation decoupling (the LSS).

The outcome is different for different matter scales, because that crucially affects the time of horizon exit (during inflation) and re-entry (after inflation). One should note here that the ordering of events given here depends on the relative energy densities of matter, radiation, and dark matter: if they were different the relevant epochs might be different. So the analysis given here is model dependent to that extent; it may differ with different numbers for these densities. But it is in accord with the best present estimates of how things are.

We now look at these epochs in turn, from inflation on, for the physical scale of the MW region. The epoch before inflation is not well understood, but also has a minimal effect on what occurs after inflation, because the exponential expansion during inflation wipes out memory of conditions before. In this sense the start of the history of the known part of the universe is during the inflationary epoch. The preceding era is both unknown, and largely irrelevant (given that it set up the conditions for inflation to occur).

A summary of the situation is given in Figure 1, which traces the effective domain of influence for the MW region, and its matter horizon, back to earlier and earlier times. These results are all based on standard calculations, such as those summarized in Dodelson (2003).

3.2   Inflation and horizon reentry

To begin with during inflation, the MW perturbations were continuously generated inside the effective horizon H^{-1}, but they would have quickly left H^{-1} during the inflationary period itself and become "frozen in".

Before the MW perturbations re-entered "the horizon" (the Hubble scale) at t_{hor}, very soon after reheating at the end of inflation, there was no possibility of causal contact with other perturbations through acoustic oscillations – that is, at t_{exit}, they quickly left "the effective horizon" H^{-1} after their generation during inflation. Before leaving, no correlations or causal connections that would persist into the classical regime were established. Remember that before t_{exit} the perturbations are purely quantum, and as such have no realized classical status except as probability waves. [7] So there is no expansion of the realistic causal domain due to acoustic waves or light-like waves earlier than t_{hor}.

3.3   From horizon reentry to CDM decoupling

When the universe reheated at the end of inflation, not only would the universe have been radiation dominated, but also

---

[7] Once the perturbations leave the horizon, they do, in fact, become classical, or at least quasi-classical. See Linde 1990, Liddle and Lyth 2000, and Kiefer and Polarksi 2008.

there would have been a certain period of time thereafter during which the dominant dark-matter particles, as well as the baryons, would have been coupled to the radiation and other matter in the universe by interactions other than gravity. Under these conditions, the relativistic sound speed $c_s = c/\sqrt{3}$ determined the maximum limit of this dominant causal influence at that epoch. This determines the effective horizon of communication at that time. Thus, from horizon re-entry at t_{hor}, shortly after the end of inflation, until t_{eq} – when the universe became matter-dominated – causal influences propagated and correlated the density perturbations within a much larger region than given by the matter horizon, through the acoustic waves travelling at c_s. No influences, except for gravitational waves and neutrinos will travel over large distances faster than c_s. Photons always have the velocity c, but in a dense ionized plasma electromagnetic information will be diffusing outward from a source at considerably less than that because the photon mean free path is so short.

Gravitational waves and some decoupled weakly interacting relativistic particles (e. g., neutrinos) would have higher velocities than this, but would not, in the ΛCDM cosmologies, have any significant effects on perturbation size or growth. Though the MW CDM perturbations would remain intact and grow between horizon re-entry just after inflation/reheating and CDM decoupling, there would have been little or no coalescence or merging – they were still very tenuous, in the linear regime, and dominated by and expanding with the Hubble flow. However, because of the coupling of matter to radiation they were subjected to coherent CDM acoustic oscillations, which propagated through the medium at c_s. There will also be some matter diffusion, but at a much slower speed: there will be very little spreading due this cause(the universe is expanding very fast then, and that will certainly dominate diffusion effects).

Since they are relatively small-scale, the MW CDM perturbations re-enter H^{-1} relatively early, at t_{hor} during the radiation-dominated era. While outside H^{-1} after the end of inflation, they were "frozen in" but still able to grow in size as $a^2(t) \propto t$, since the radiation pressure was not effective over superhorizon distances (i. e., they were not causally self-connected). Thus, in summary, as soon as each MW-size fluctuation cluster – and before that each of its components – re-entered the effective horizon at these very early times during the radiation-dominated era, it was in internal causal contact, and growing logarithmically. Before re-entering the effective horizon but after the end of inflation, it would not have been in causal contact, but growing as $a^2(t) \propto t$. However, the matter horizon extends virtually unchanged all the way back from decoupling at the LSS to reheating at the end of inflation, when the scalar-field fluctuations generated by inflation are transformed into matter-radiation density perturbations. This is the origin of the matter that exists in the MW domain.

3.4   From CDM decoupling to matter domination

Decoupling of the dark-matter itself from the other particle species depends crucially on the particular dominant dark-matter particle, and on its interactions with other particles. As long as its interaction rate Γ with other particle species is greater than H, thus keeping it in equilibrium with the

universe, it will be coupled. When $\Gamma < H$ it effectively decouples from the system – it evolves separately from the other particle species (see Kolb and Turner 1990, p. 115, pp. 119-130). Unfortunately, we know neither the identity nor the distinguishing characteristics of these CDM particles. Thus, we really cannot determine when this decoupling occurs. Since the perturbations are CDM, though, we do know that the particles must have decoupled when the they were non-relativistic – that is when $m_{CDM} >> T_d$, where T_d is the temperature at which $\Gamma_{CDM} \leq H$. This means that, even though they decoupled long before z_{eq}, they are massive enough so that they have relatively small random thermal velocities. This is characteristic of all CDM.

However, once the CDM particles decouple, their perturbations are subject to damping by free-streaming. CDM particles can stream out of the overdensities, causing them to evanesce. This can only happen for short length-scale inhomogeneities. Kolb and Turner (1990, pp. 352-353) calculate the general free-streaming comoving length scale for such particles as

$$\lambda_{FS} = 0.2 Mpc(m_X/keV)^{-1}(T_X/T)[ln(t_{eq}/t_{nr}) + 2], \qquad (1)$$

where m_X is the mass of the CDM particle in keV, T_X is the temperature of the CDM particles, T is the temperature of the photons, t_{nr} is time at which they become non-relativistic, and t_{eq} is the time of radiation-matter equality. Generally speaking for CDM particle candidates T_X will be significantly less than T, and t_{nr} will be significantly less than t_{eq}.

Thus, until we know more about the dominant CDM candidate, we cannot reliably establish the length-scale below which our CDM perturbations will be significantly damped by free-streaming. It is very likely, though, that the primordial CDM perturbations of the size that triggered the early formation of what became the MW remained intact. Furthermore, they certainly gradually accumulated substantially more mass by accretion and through mergers with other nearby perturbations. We shall presume this in the rest of this paper. If they did not remain intact, then it is not possible to explain the plethora of structure we see on small (galaxy-sized) scales, given the evidence for bottom-up galaxy formation. A complementary top-down structure formation process for such small-scale, MW-size perturbations is not supported by the evidence. From equation (1), we can see that it is not hard to account for persisting CDM structures on scales $\approx$ 1 Mpc, as long as m_X is not much less that 1 keV.

After CDM decoupling but before photon decoupling, the baryons were still tightly coupled to radiation (photons to electrons via Thomson interaction, protons to electrons via Coulomb interaction). Thus the baryons cannot collapse because of radiation pressure, whereas CDM is not so constrained. The complication here is that between CDM decoupling at t_{CDM} and matter-radiation equality later at t_{eq}, the universe was dominated by radiation coupled with the baryons. Though the CDM was decoupled from that mixture, it was dominated gravitationally by the radiation and therefore forced to move through gravitational coupling with baryon-radiation acoustic oscillations. Once the universe became matter dominated (at t_{eq}), the CDM gravitationally dominated the baryon-radiation fluid, and the decoupled CDM perturbations modulated the baryon-radiation acoustic oscillations.

In the period right after t_{CDM} – certainly while radiation was still in control(until t_{eq}) – CDM perturbations which had already re-entered the effective dynamic horizon H^{-1} did not grow significantly (see, for instance, the detailed treatment of Dodelson 2003, especially Section 7.3). Fluctuations begin to grow approximately logarithmically, with a slightly more enhanced rate of growth as the matter dominated regime is approached, at t_{eq}. This early relative suppression of small length scale perturbations has been detected in the mass spectrum. It is possible that before CDM decoupling diffusion altered the perturbations, but we have not taken it into consideration, because the effects would not be large.

3.5 From matter domination to photon decoupling

For our universe, with $\Omega_{m0} = 0.3$, the redshift z_{eq} of radiation-matter equality would be roughly at $1 + z_{eq} \approx 3500$. This is considerably before recombination at $z_{rec} \approx 1100$. The age of our universe at t_{eq} when the matter and radiation densities are equal, assuming our universe has $\Omega_{\Lambda 0} \approx 0.7$, $\Omega_{tot} = \Omega_m + \Omega_{rad} + \Omega_\Lambda = 1$, was $t_{eq} \approx 8.9 \times 10^3$ years.

If our universe were not expanding, causal influences could effectively propagate a distance $c_s t_{eq} = (c/\sqrt{3})8.87 \times 10^3 \; years = 5.12 \times 10^3 \; lightyears = 1.57 \times 10^{-3} \; Mpc$ by $t = t_{eq}$. [8] However, our universe *is* expanding, and so the effective domain of influence $d_s(t)$ at $t = t_{eq}$ will really be as given by :

$$d_s(t_{eq}) = \frac{a(t_{eq})}{\sqrt{3}} \int_0^{t_{eq}} dt'/a(t'). \qquad (2)$$

Using the fact that in the radiation-dominated era $a(t) \propto t^{1/2}$ (even if $\Omega_{\Lambda 0} = 0.7$, $\Omega_{\Lambda eq}$ will be extremely small, as can be easily shown), we find that

$$d_s(t_{eq}) = 2ct_{eq}/\sqrt{3} = 3.14 \times 10^{-3} \; Mpc. \qquad (3)$$

As a co-moving distance, relative to distances at our redshift now, our result is $d_s(t_{eq})_0 = 11.08 \; Mpc$. Comparing this with the co-moving length scale of 1.6 Mpc for our MW cluster of perturbations at t_{rec}, which is later than t_{eq}, we confirm that MW-size fluctuation clusters were already well within the cosmological causal limit at t_{eq}. It was also well within the Hubble scale H^{-1}, which is often referred to as "the horizon" in terms of dynamical dominance.

Once the MW perturbations enter the matter-dominated regime ($t > t_{eq}$), they grow as $a(t) \propto t^{2/3}$, much more rapidly than logarithmically (we are assuming an almost flat universe). There are very slight modifications to this basic picture due to the presence of baryons and dark energy. This growth, of course, presumes that that MW CDM perturbations are stable against free-streaming, as indicated

---

[8] Here we are assuming that there is no substantial contribution from the extremely brief period between the Big Bang and the end of inflation. Since the sound speed for a scalar field will be c, anyhow, that will not make any difference.

above. This is why confirmation that the mass of the dominant CDM particle is large – to explain the survival and early growth of smaller CDM fluctuations – is important to verify. After recombination, as we have already mentioned, the baryons are then free of the radiation, and can follow these CDM perturbations, falling into their potential wells.

But what about the remaining baryon-acoustic oscillations before recombination and photon-baryon decoupling when the universe is dominated by CDM? During that period long-range correlations can be established via those sound waves – but now modulated, as we have just said, by the presence of the developing CDM potential wells. Since all the growth in those perturbations and their gravitational influence is still in the linear regime, those baryon-acoustic oscillations will not be trapped in those perturbations and will communicate across them. Thus, we need to extend the growth of the effective domain of influence forward to the t_{rec}, the time of photon-baryon decoupling and recombination. This causal limit, which has often be referred to as the "sound horizon" or the "acoustic horizon" has been discussed extensively in the literature since at least 1970 (see Peebles and Yu 1970; Sunyaev and Zeldovich 1970; Eisenstein and Hu 1998; Dodelson 2003; and Desjacques 2008, and references therein). It is represented in Figure 1 by the diagonal broadening of the domain of influence from the matter horizon at t_{rec}, not just t_{eq}, all the way back to t_{hor}. This means that we should find density correlations in the large-scale structure on that scale – and indeed we do, as enhancements in the galaxy and cluster-of-galaxy correlation function at about 105 h^{-1} Mpc (Eisenstein, et al. 2005; Cole, et al. 2005; Estrada, et al. 2008; Desjacques 2008). This corresponds, as is very well known also, to the the position of the first peak in the temperature anisotropy measurements of the CMB (see, for instance Hu, et al. 2001 and Doran and Lilley 2002, and references therein).

Theoretically, how is that scale explained? Eqn.(3) shows that $d_s(t_{eq}) = 2ct_{eq}/\sqrt{3}$ where we are assuming that horizon re-entry for the MW perturbation occurs just after inflation ends at $t \approx 0$, relative to t_{eq}. To this has to be added the extra distance due to the acoustic wave propagation still being effective during the early matter-dominated epoch from t_{eq} to t_{rec}. This will be

$$d_s(t_d) = \sqrt{3}c(t_{rec} - t_{rec}^{2/3}t_{eq}^{1/3}), \qquad (4)$$

where $t_d \equiv t_{rec} - t_{eq}$. Then the distance over which density correlations were induced by the acoustic oscillations between horizon entry and recombination is simply

$$d_s(t_{rec}) = d_s(t_{eq}) + d_s(t_d). \qquad (5)$$

This distance defines a correlation length at horizon re-entry for the perturbations immediately surrounding the MW perturbations – and constitutes the realistic domain of influence for the MW perturbations at that time. It gives the limit out to which the dominant causal influences have significantly impacted and connected that region and the objects in contains. It is much larger than the matter horizon, which specifies the limit from which material has been permanently added to our region.

In principle and in practice, as we just mentioned above, this effective domain of influence is observable now in intermediate scale density correlations in the nearby universe, the "baryon acoustic oscillations" recently detected (Eisen-

stein, et al. 2005). $d_s(t_{eq}) \approx 3.14 \times 10^{-3} Mpc$, which translates into a comoving distance (relative to the present) of 11.08Mpc. Using the relationship between t_{rec} and z_{rec} (see Kolb and Turner 1990, p. 80), we find from the above that $d_s(t_d) = 9.13 \times 10^{-2}$ Mpc, or 100.4 Mpc, comoving. Thus, the comoving scale of the effective causal or sound horizon $d_s(t_{rec})$ is about 111 Mpc. This corresponds to a spherical harmonic index of $l \approx 80$, or about 38 arcmin., which is an easily observable angular scale in CMB data.

3.6 The effective domain of influence

Thus, in the standard ΛCDM universe – or in any one not too different from it – the topography of local causal domains significant for our cosmic region illustrated in Figure 1 involves the narrow core of the matter horizon extending from the MW all the way back down to the generation of its components in the reheating at the end of inflation, with a comoving size of about 2Mpc. Shrouding this matter horizon is the realistic domain of influence, expanding out to a comoving size of approximately 110 Mpc, through the effects of CDM-radiation and baryon-radiation acoustic waves between t_{hor} and t_{rec}. This more extensive domain which characterizes the causal history of our locale before t_{rec} does not add material to the MW perturbations themselves or significantly alter the world-lines along which they flow, but rather correlates them with other clusters of perturbations due to the causal influence of the acoustic oscillations on the CDM and the baryons before recombination and baryon decoupling.

The effective domain of influence is much smaller than both the particle horizon and the visual horizon, firstly because its broadening out as we go back into the past from the LSS starts from the scale of the matter horizon at the LSS, not the scale of our past light cone; and secondly, because it broadens out at an effective speed of $c/\sqrt{3}$, rather than c. Furthermore it effectively stabilizes at horizon re-entry, remaining the same size until the start of the classical domain where we can sensibly follow its evolution.

4 THE PRESENT AND THE FUTURE

So far in this paper we have, from the perspective of the past history of the universe, considered the MW as our local dynamical region, and its extent as our present matter horizon. However, as we have emphasized, the MW itself is continuing to grow by accreting material, including smaller systems, from its environment, and is gravitationally interacting with nearby and more distant galaxies and clusters – most clearly, the members of the Local Group, and the Virgo cluster. Taking these present interactions with other systems into account, as well as those in the distant future, can we physically and quantitatively characterize, or define, the growth of the matter horizon and the effective domain of dependence as the universe evolves into the future? This would give us the dynamical limit of our cosmic locale.

We have already briefly mentioned several possible definitions. As we have emphasized, as long as our universe continues to be dominated by ΛCDM, with no extreme exotic surprises, as we have been assuming, there are vast regions of it which will never alter, or significantly influence, our own

locality – whose world lines will never approach or intersect our own world line. There are regions at a sufficient distance from us – but not at an extraordinary distance from us – which are now expanding away from us. And there are others, somewhat further out, which will never collapse towards us – no matter how far into the future we go.

Motivated by these general observations about the relationship between our locality and the universe, we define both a present domain of influence and a future matter horizon. This is related to the idea of a finite infinity surrounding us, see Ellis (1984) and Cox (2007). The way they are related to each other is indicated in Figure 2.

4.1 The future matter horizon

The *future matter horizon* is, ideally, the spherical shell [9] at the radial distance r_{oh} from any point q_0 – e. g. our own location – beyond which no world-lines will ever converge towards q_0, beyond which nothing will ever collapse towards us.

In working out the criterion for the future matter horizon r_{oh} we can apply the approach Padmanabhan (1993, 275-276) uses to work out in Newtonian approximation the condition for the eventual collapse of an overdense region in an expanding universe. The condition itself is on $\bar{\delta}_i$, which is the average overdensity of the region out to a radius r_i at a particular time t_i relative to the background cosmic density at that time. This key parameter is given by

$$\bar{\delta}_i = \frac{3}{r_i^3} \int_0^{r_i} \delta_i(r) r^2 dr, \tag{6}$$

where $\delta_i(r)$ is the local density contrast. Essentially, as we shall see, the future matter horizon r_{oh} is the value of r_i in equation (6) – in both the denominator and in the upper limit of the integral – which gives the value of $\bar{\delta}_i$ which is just low enough so worldlines at r_i will never converge towards us. It is the boundary between the region around us which is collapsing or will eventually collapse, and the region beyond, which will never approach us and continues to expand forever. In implementing this criterion we first must determine what value of $\bar{\delta}_i = \bar{\delta}_{oh}$ insures that, and then from observational measurements of $\delta_i(r)$ determine at what value $r_i = r_{oh}$ that value is reached. Padmanabhan (1993) worked out the first part of this problem in general for a $\Lambda = 0$ FLRW universe. Here we shall do the calculation for our case – where $\Lambda \neq 0$.

Assuming spherical symmetry and using the Newtonian approximation, we have the energy E for a given overdense region in an FLRW universe

$$E = \frac{1}{2}(\frac{dr_i}{dt})^2 - \frac{GM}{r_i} + \frac{\Lambda}{3}r_i^2, \tag{7}$$

where r_i is the radius of the region, M its mass, and Λ is the vacuum energy (cosmological constant). The condition for eventual collapse is obviously $E < 0$.

---

[9] It is very clear that the MW is *not* at the center of our dynamical local domain – with Virgo-centric infall and the motion of our neighboring clusters of galaxies towards the Great Attractor. However, here we simplify our analysis to spherical symmetry to formulate and examine the suitability and implications of the basic concepts.

We cannot use Padmanabhan's simplest calculation, since it assumes that $\bar{\delta}_i$ is small enough so that the boundary of the region at r_i is still expanding with the Hubble flow. This will certainly not be true in our case, with $\Lambda \neq 0$, as we shall see. Thus, we use his generalization (outlined in Padmanabhan 1993, p. 320, Exer. 8.1) to obtain

$$\bar{\delta}_{oh} \geq (1 + \frac{v_{oh}}{H_0 r_{oh}})^2 \Omega_m^{-1} + \frac{2\Omega_{\Lambda 0}}{\Omega_{m0}} - 1, \tag{8}$$

where we are considering the situation now ($t = t_0$) at r_{oh}, and where $v_{oh} < 0$ is the peculiar (radial) velocity there due to the local gravitational potential within r_{oh}.

Further, we know from the detailed work of an number of researchers (see Lahav, *et al.* 1991, and references therein) that v_i is related to the $\bar{\delta}_i$ within the region by

$$v_i = \frac{1}{3}H_0 r_i f \bar{\delta}_i, \tag{9}$$

where f is a function of Ω_m and Ω_Λ. For our time now it is given (Lahav, *et al.* 1991) by

$$f_0 \approx \Omega_m^{0.6} + \frac{1}{70}\Omega_\Lambda(1 + \frac{1}{2}\Omega_m). \tag{10}$$

Thus, combining equations (8), (9) and (10), we solve a quadratic equation to obtain, for our values of Ω_{m0} and $\Omega_{\Lambda 0}$,

$$\bar{\delta}_{oh} \geq 4.0. \tag{11}$$

This means that for all $r_i < r_{oh}$, such that $\bar{\delta}_{oh}$ is given by equation (8), or in our case by equation (11), the world lines will eventually undergo collapse, even if they are still expanding now.

With the result (9), if we also know $\delta_i(r)$ throughout the region bounded by r_{oh}, we can use equation (6) to find the value of r_{oh} itself. Unfortunately, we do know $\delta_i(r)$, or its non-spherically symmetric generalization, far enough out to determine r_{oh} reliably. But this should certainly be possible to estimate in the near future.

One clear consequence of this simple calculation is that r_{oh} in our observable universe will *not* be the boundary beyond which the matter flow lines are moving with the Hubble flow. Furthermore, it is also clear that even for a range of values of $r > r_{oh}$, the geometry is is not yet perturbed FLRW. That boundary is still farther out, at r_b such that $\bar{\delta}_b < 1$. This is also obvious from the peculiar velocity at r_{oh}, $v_{oh} = -0.67v_{red}$, where $v_{red} \approx H_0 r_{oh}$ is the "redshift velocity" at r_{oh}. Thus, outside r_{oh} there will be a fairly large region which, though influenced by the Hubble expansion and fated to expand forever, is at present significantly influenced by the peculiar velocity due to $\bar{\delta}_{oh}$.

4.2 The Present Domain of Influence

The *present domain of influence* r_{ih} is defined by the surface at the largest distance from our position where the matter flow lines are moving towards us (collapsing) at our time now, $t = t_N$. Beyond that radius they are still dominated by cosmic expansion. It is the radial distance from q_0 beyond which expansion from us is still occurring, and inside of which there is overall collapse of matter world lines towards us. We define r_{ih} both because it gives a clearer dynamical definition of the actual local domain of influence at any given time and because it can be defined in the case of a closed universe, whereas r_{oh} cannot be.

As we have already indicated, the boundary of the present domain of influence r_{ih} is the distance now to the shell of material surrounding q_0 which is just about to begin collapsing. This is the extent of the region centered on q_0 in which world lines are now converging. Outside r_{ih} they are presently influenced by the cosmic expansion, and still diverging. The matter in this region, obviously, will eventually affect our locale, and will do so sooner than that at radii between r_{ih} and r_{oh}. The criterion for determining r_{ih} is quite simple, and somewhat similar conceptually to that for determining r_{oh}. Essentially, at r_{ih} the peculiar radial velocity v_i, which is induced by the overdensity of matter $\delta_j(r)$ within the region exactly cancels the velocity $H_0 r_{ih}$ due to the cosmic expansion of the universe. Here $r_{ih} = x_{ih} R_0$ is the actual physical distance to the inner realistic horizon, where x_{ih} is the comoving distance to it and R_0 is the scale factor of the universe. Thus, at r_{ih} we have

$$H_0 r_{ih} = v_{ih}. \tag{12}$$

Applying equation (9) above, we find that

$$\bar{\delta}_{ih} = 3f^{-1}, \tag{13}$$

where, as before, $\bar{\delta}_{ih}$ is given by equation (6) with $r_i = r_{ih}$. With again $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ and using equation (10), equation (13) gives

$$\bar{\delta}_{ih} = 6.1. \tag{14}$$

At smaller radii, where the world lines are converging, $\bar{\delta}_i$ will be larger than this, and at larger radii, where the shells will be feeling the cosmic expansion to some extent at least, it will be smaller. Obviously, this result is consistent with our result for r_{oh}, since $\bar{\delta}_{ih} > \bar{\delta}_{oh}$, as it must be, for $r_{ih} < r_{oh}$. Once again, of course, in order to determine r_{ih} itself from our calculation of $\bar{\delta}_{ih}$, we need to to know the local density contrast function $\delta_i(r)$ within that region and then solve equation (8).

In principle, r_{ih} could be observationally determined, by finding the boundary where the total redshift (cosmological + peculiar-velocity) of relatively nearby galaxies and clusters of galaxies is zero. This would enable the direct determination of r_{ih} at the look-back time corresponding to that redshift. r_{ih} now, which is not directly determinable by observation, could then be extrapolated from that result. Because of all the local motions within the various sub-systems, the first step in this program would undoubtedly require a great deal of careful observational and statistical work.

5 CONCLUSION: THE DOMAINS OF INFLUENCE

The effective domains of influence defined here – the matter horizon, the effective causal horizon, the present domain of influence, and the future matter horizon – are obvious, important, but often unarticulated, characteristics of our universe. The part of space-time that significantly influences us is a very small part of our causal past, as is clearly shown in Figure 1. On intermediate and large scales, the observable universe is so well modelled by linearized gravitational effects, and the galaxies and clusters of galaxies are so spread out, that conditions at even moderate distances do not interfere with the local physics of the solar system or of our

Galaxy. However, paradoxically, this is because conditions are set up in the universe so that isolated systems exist and function more or less independently of distant regions. This need not have been true.

This might be described as an effect of the geometry of our universe being so close to FLRW on large scales. Interestingly enough, we could say from a complementary perspective that this property *is* highly relevant to local physics, because it insures that relatively isolated systems like ours are able exist and evolve on their own. Cosmology establishes the overall context within which that becomes possible! And certainly, on the standard understanding, inflation was crucial to insuring that our universe was homogeneous and diffuse enough to manifest this localization property.

To repeat the cautionary note we started with: there are other exotic possibilities for the nature of matter in the universe than those assumed here, for example we do not know for certain that the dark energy does not have a large velocity relative to the baryon rest frame. A relativistic "wind" of dark energy could significantly alter the "horizons" discussed in this article (Maroto 2006, Jimenez & Maroto 2008). However this is not the standard view; if such physics were to eventually turn out to be the case, what is presented here would have to be modified accordingly. Given the standard context, the significant causal domains for our local region are as outlined in this paper and summarized in Figure 1.

Two indirectly related issues are connected with this relative insensitivity of local physics to other locales at cosmic distances. The first is that we can only measure the integrated effect of anisotropies on the velocities of matter in our region. This is true also of local tidal effects due to large lumps of matter at large distances from us. In these cases the influence of each shell of material at large distances will cancel – unless the mass distribution is very anisotropic.

Secondly, gravitational influences *are* felt at large distances, even outside the conventional visual, particle or event horizons, via the "Coulomb" gravitational interaction (Ellis and Sciama 1972). Gravity is a long-range force and that N-body dynamics is chaotic so that the influence of the precise distribution of matter in the large-scale structure is expected to be significant. It is exactly for this reason that numerical simulations of clusters must include a much larger volume than the cluster itself to be "realistic". Because they are closely associated with the mass-energy-momentum conservation equations, and therefore with the constraint equations of general relativity (those which must be fulfilled on each space-like slice of space-time), these effects are instantaneous. That is why they can be felt beyond horizons. However, they cannot transmit new information: the gravitational influence of a large distant system is unchanging, because its mass-energy must be conserved. Because the external field is almost isotropic, these effects will not be significant on larger scales than those associated with the Baryon Acoustic Oscillations, and they do not change our conclusions.

These two features of gravity reinforce the very special large-scale character of our universe, and the protection is offers to the dynamical integrity of local regions.

In summary:

1.) Since the time of decoupling of matter and radiation, the domain that really matters as far as local physics

is concerned is the cluster of linear perturbations which became the MW, with co-moving size (relative to our present time) between 1.5 and 2.3 Mpc. This is much smaller than any discernible small-scale anisotropy in the CMB. In fact, a perturbation of that size would be unobservable in principle through CMB observations, because it is of a length scale smaller than the thickness of the last-scattering surface itself. Thus, we cannot detect or study perturbations of this scale in the CMB.

2.) Once the CDM and the baryons decouple from the radiation during the radiation-dominated and the early matter-dominated eras, the local physics is dominated by the CDM matter flow along nearby world-lines, rather than by electromagnetic or acoustic waves. This is primarily due to the near homogeneity of the cosmic background, and the lack of direct influence of conditions at even moderate distances from a given locale. This becomes an even more dominant feature after radiation-matter equality, and later after recombination.

References

Carlberg, R. G., et al. 2000. Ap. J. 532, L1.

Cho, A. 2008. Science 319, 47.

Cole, S., et al. 2005. Mon. Not. R. Astron. Soc. 362, 505.

Côté, S., Carignan, C., & Freeman, K. C. 2000. Astron. J. 120, 3027.

Cox, D.P.G. 2007. Gen. Rel. Grav. 39, 87.

Desjacques, V. 2008. arXiv: astro-ph/0806.0007v3.

Dodelson, S. 2003. *Modern Cosmology*, Academic Press, 440 pp.

Doran, M. and Lilley, M. 2002. Mon. Not. R. Astron. Soc. 330, 965.

Eisenstein, D. J. and Hu. W. 1998. Astrophy. J. 496, 605.

Eisenstein, D. J., et al. 2005. Astrophys. J. 633, 560.

Ehlers, J., Prasanna, A. R., and Breuer. R. A., 1987. Class. Quantum Grav. 4, 253.

Ellis, G. F. R. 1984. In *General Relativity and Gravitation*, Ed B Bertotti et al, Dordrecht: Reidel, 215.

Ellis, G. F. R. & Sciama, D. W. 1972. "Global and Non-Global Problems in Cosmology," in *General Relativity: Papers in Honour of J. L. Synge*, L. O'Raifeartaigh, editor, Oxford, Clarendon Press, pp. 35-59.

Ellis, G. F. R. and Stoeger, W. R. 1988. Class. and Quantum Grav., 5, 207.

Estrada, J., Sefusatti, E., and Frieman, J. A. 2008. arXiv: astro-ph/ 0801, 3485.

Hawking, S. W. and Ellis, G. F. R. 1973. *The Large Scale Structure of Space Time*. Cambridge University Press.

Hu, W., Fukugita, M., Zaldarriaga, M., and Tegmark, M. 2001. Astrophys. J. 549, 669.

Ibata, R. A. & Lewis, R. 2008. Science 319, 50.

Jimenez, J.B. & Maroto., A.L. 2008. arXiv:0811.3606v3.

Kiefer, C. and Polarski, D. 2008. arXiv: astro-ph/0810.0087v1.

Kolb, Edward W., and Turner, Michael S. 1990. *The Early Universe*, Addison-Wesley Publishing Co., 547 pp.

Klypin, A., Hoffman, Y., Kravtsov, A. V., and Gottlöber, S. 2003. Ap. J. 596, 19.

Lahav, O., Lilje, P. B., Primack, J. R., & Rees, M. J. 1991. MNRAS 251, 128.

Liddle, A. R. and Lyth, D. H. 2000. *Cosmological Inflation and Large-Scale Structure*, Cambridge, Cambridge University Press, pp. 177-185.

Linde, A. 1990. *Particle Physics and Inflationary Cosmology*, 4th Printing (1993), Harwood Academic Publishers, Chur, Switzerland. 362 pp.

Maartens, R., Ellis, G. F. R., and MacCallum, M. 2007.

Maroto, A.L. (2006). JCAP 0605:015.

Murali, Chigurupati, et al. 2002. Ap. J. 571, 1.

Padmanabhan, T. 1993. *Structure Formation in the Universe*, Cambridge University Press, 483 pp.

Peebles, P. J. E. 1993. *Principles of Physical Cosmology*, Princeton University Press, 718 pp.

Peebles, P. J. E. and Yu, J. T. 1970. Astrophy. J. 162, 815.

Rindler, W. (1956). *Mon Not Roy Ast Soc* **116**, 662.

Sunyaev, R. and Zeldovich, Ya.-B. 1970. Astrophys. J. Suppl. Ser. 7, 3.

Tipler, F J , Clarke, C J S, and Ellis, G F R (1980): "Singularities and Horizons: a review article". In *General Relativity and Gravitation: One Hundred years after the birth of Albert Einstein*, Vol. 2, Ed. A. Held (Plenum Press), 97-206.
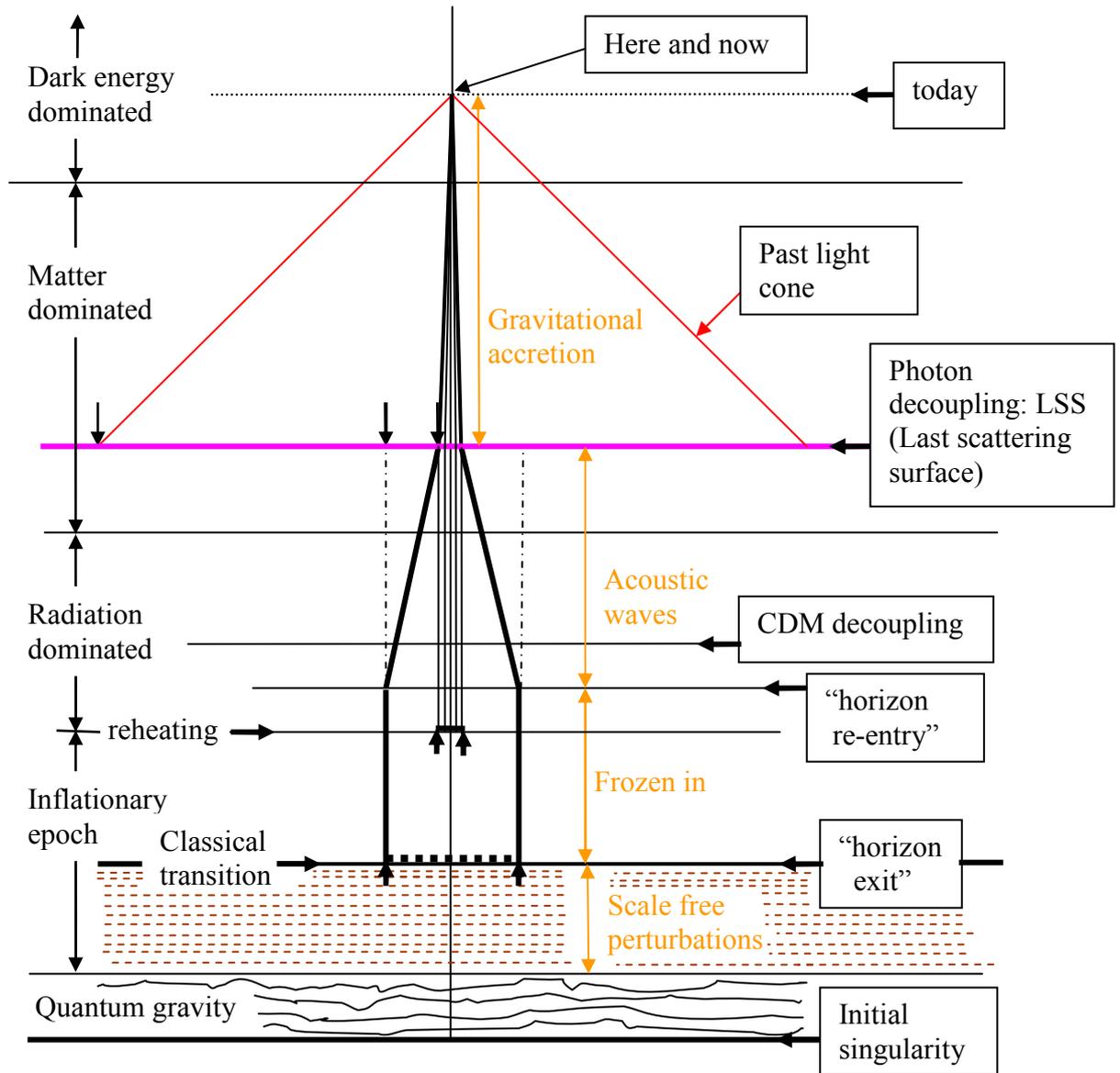
The three downwards pointing arrows on the LSS (photon decoupling) show (left to right) the size of the visual horizon, and the comoving size of the effective domain of influence between the start of the inflationary era to horizon re-entry in the radiation dominated era, and at decoupling of matter and radiation (NB: not to scale!). The two inner upward pointing arrows show the size of the matter horizon at the origin of the MW matter when reheating takes place at the end of inflation.

Figure 1: Effective Domains of influence: Space time diagram.

Caption: The co-moving size of the effective domain of influence (dark broken line stretching down from "here and now") is slightly increasing as one goes back into the past in the matter dominated era, as matter accretes onto the central mass. It increases as $c/\sqrt{3}$ in the radiation dominated (acoustic oscillation) era until "horizon exiting", when it becomes constant until horizon exit, which is where the relevant perturbations become classical. The visual horizon size is given by the past light cone intersecting the last scattering surface; the particle horizon size would be attained by extending the past light cone to the start of the quantum gravity era.
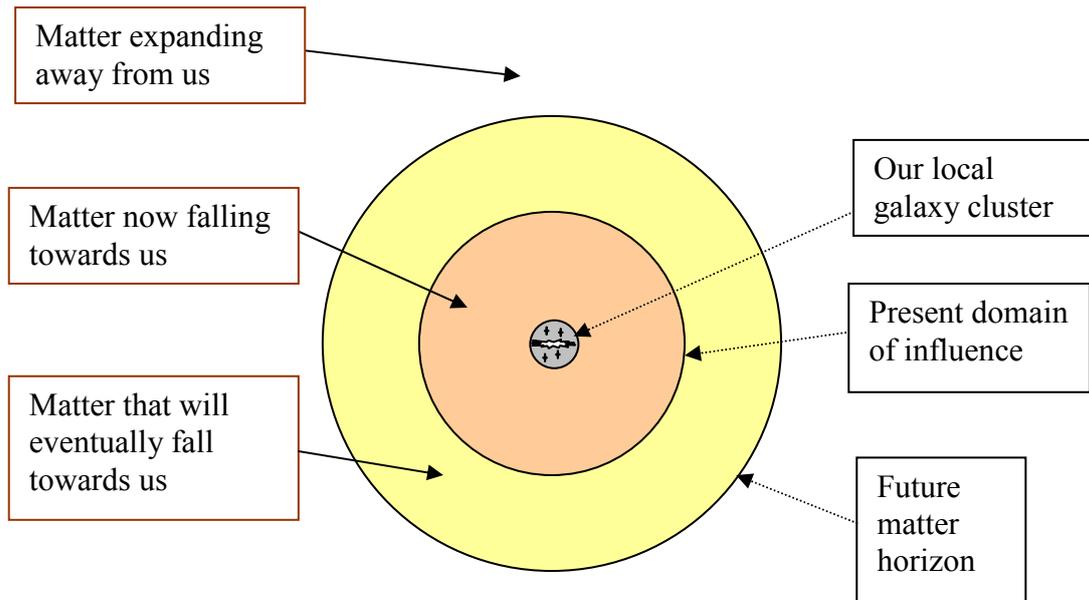
Figure 2: Realistic causal influences: spatial diagram

Caption: The present domain of influence is the limit of region around our galaxy where the gravitational influence of the local group is currently causing the cosmological expansion to turn around. The future matter horizon is the limit of region around our galaxy where the gravitational influence of the local group will eventually cause a re-collapse